

Measuring the Elementary Charge by Way of the Millikan Oil Drop Experiment

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Abstract

This experiment aimed to measure the elementary charge e by using the Millikan Oil Drop Experiment, which demonstrates the discrete, quantized nature of electric charge. After atomizing oil droplets and observing their behavior between charged capacitor plates, the terminal velocities before and after applying an electric field were measured. From these measurements, the charge of each droplet was calculated through an analysis of the forces acting on the droplets. These charges were then shown to cluster around discrete values, allowing a weighted-least-squares regression to be performed. With uncertainties carefully propagated throughout the calculations, the experiment yielded a value of $e = (-1.60 \pm 0.25) \times 10^{-19}$ C, which is in excellent agreement with the currently accepted value of $e = -1.602177 \times 10^{-19}$ C. This result confirms the quantized nature of electric charge and demonstrates the effectiveness of the experimental method.

1 Introduction

The purpose of this experiment is to demonstrate the quantum/discrete nature of electric charge, measure the value of the elementary charge e , and correctly propagate uncertainty throughout the calculations to determine the uncertainty of the result.

The measurement of the elementary charge is a cornerstone of modern physics, as it provides direct evidence of the quantized nature of electric charge. This experiment, first performed by Millikan and Fletcher, was crucial in advancing our understanding of atomic structure, and contributed to the eventual acceptance of quantum theory.

The principles demonstrated here are directly applicable to modern technology. For example, the quantization of electric charge is essential in fields such as semiconductor physics, where understanding charge carriers in devices like transistors and capacitors is critical for the design of integrated circuits.

General learning goals of the experiment and analysis include developing student independence,

increasing laboratory proficiency, and introducing methods in data analysis and statistics.

1.1 Background

By the end of the 1890's, much of what was known about electricity and magnetism could be explained on the basis that charge is continuous, and the existence of subatomic particles was not universally accepted. However, while experimenting with cathode rays in 1897, J.J. Thompson discovered negatively charged "corpuscles" with a mass of about $\frac{1}{1837}$ that of a hydrogen atom.

In order to further demonstrate the discrete nature of electric charge, and determine the value of the elementary charge e , Robert Millikan and Harvey Fletcher designed and performed what is now known as the Millikan Oil Drop Experiment. This experiment, performed in 1909, earned Millikan the Nobel Prize in Physics of 1923[5].

1.2 Methods

To accomplish the goal, an oil-drop experiment will be performed where oil will be atomized into a fine mist and sprayed between the parallel plates of a capacitor. The droplets will then be charged, where discrete changes in their terminal velocities associated with discrete changes in their surface charge can be observed after multiple separate chargings. This concept is demonstrated in Figure 2, and a simplified experiment setup is demonstrated in Figure 1.

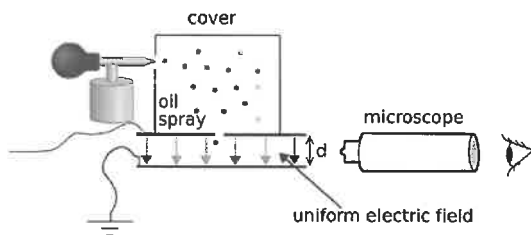


Figure 1: A simplified diagram of the experimental setup. This demonstrates that oil will be sprayed into a chamber, from which it will enter into a region between two plates of a capacitor. The application of a uniform electric field in this region will allow the gathering of data that leads to the calculation of e .

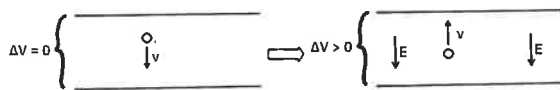


Figure 2: The basic concept behind the experiment. The terminal velocity v of a charged droplet is measured before and after a voltage difference ΔV is applied between parallel plates. A comparison then allows the charge of the droplet to be determined.

1.3 Results

This experiment and analysis yields the following value for the elementary charge e :

$$e = (-1.60 \pm 0.25) \times 10^{-19} \text{ C} \quad (1)$$

The currently accepted value for the elementary charge is $e = -1.602177 \times 10^{-19} \text{ C}$ [3]. Thus the calculated value is only 0.01 errors away from the accepted value, in excellent agreement.

Therefore, this experiment and analysis successfully show that charge is quantized, and that the methods allow for the determination of the elementary charge to within a predicted uncertainty.

2 Mathematical Analysis

The analysis shall begin with the assumption that the oil droplets are sufficiently small such that the following hold:

1. their small surface area will only stably hold a small number of electrons upon being charged through ionization of the surrounding air.
2. they have a small terminal velocity in air.
3. they are spherical and of uniform density.
4. they are dominated by the force of gravity \vec{F}_g , force of drag \vec{F}_d , and Coulomb force \vec{F}_E .

The buoyant force can be ignored because the density of the displaced air is insignificant compared to the density of oil (1.3 kg/m^3 versus 886 kg/m^3).

The experiment apparatus allows one to bombard the space with alpha particles. These alpha particles knock electrons from the air molecules, allowing those free electrons to move to the oil droplets and thereby leaving them with a surplus of electrons.

2.1 Forces

A diagram of all of the forces acting on an oil droplet can be seen in Figure 3.

2.1.1 Gravity: \vec{F}_g

The gravitational force is given by $\vec{F}_g = -mg\hat{y}$. Because perfect spheres and uniform density are being assumed, this becomes:

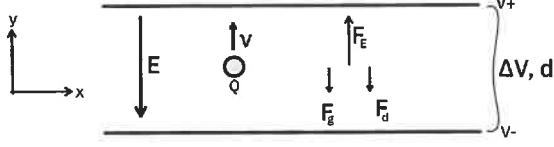


Figure 3: The forces that are acting on the droplets. The droplets are dominated by the Coulomb, drag, and gravitational forces \vec{F}_E , \vec{F}_d , \vec{F}_g .

$$\vec{F}_g = -\frac{4}{3}\pi r^3 \rho g \hat{y} \quad (2)$$

Where r is droplet radius, ρ is oil density, and g is gravitational acceleration (9.8 m/s^2).

2.1.2 Coulomb: \vec{F}_E

The Coulomb force acts on our charged droplets when there is a voltage difference between the plates that creates a uniform electric field. It is given by $\vec{F}_E = Q\vec{E}$, where Q is droplet charge and \vec{E} is the electric field.

Here a positive voltage difference is defined as the top plate having a higher voltage, as seen in Figure 3, thus a positive ΔV generates an \vec{E} in the $-\hat{y}$ direction. The magnitude of \vec{E} is given by the relationship between ΔV and a uniform field, $\Delta V = Ed$. Therefore:

$$\vec{F}_E = Q\vec{E} = -Q\frac{\Delta V}{d}\hat{y} \quad (3)$$

Where Q is the droplet charge, ΔV is the plate voltage difference, and d is the plate separation. Note: the droplets will likely be negatively charged ($Q < 0$), so \vec{F}_E will be in the $+\hat{y}$ direction for positive ΔV .

2.1.3 Drag: \vec{F}_d

Drag is a force that opposes the movement of the droplets due to momentum transfer with surrounding

air molecules. For a small sphere moving in a viscous fluid, this force is given by Stokes' Law[6]:

$$\vec{F}_s = -6\pi\eta_a r \vec{v} \quad (4)$$

Where η_a is the viscosity of air, r is the droplet radius, and \vec{v} is the droplet velocity.

However, for particles between the continuum regime and free molecular flow regime ($1\mu\text{m} - 15\mu\text{m}$; the selected droplets will be $1\mu\text{m} - 2\mu\text{m}$), a correction for non-continuum effects using the Cunningham Correction Factor is needed[2], $\vec{F}_d = \frac{\vec{F}_s}{C}$, where:

$$C = 1 + \frac{\lambda}{r} \left(A_1 + A_2 e^{-\frac{2A_3 r}{\lambda}} \right) \quad (5)$$

Where λ is the mean free path of air, $A_1 = 1.257$ for air, $A_2 = 0.400$ for air, $A_3 = 0.55$ for air, and r is the droplet radius.

The mean free path of air is around $64\text{nm} - 68\text{nm}$ [4], while the selected droplets will be $1\mu\text{m} - 2\mu\text{m}$ in radius. This gives a value in the exponential of Equation 5 of:

$$e^{-\frac{2A_3 r}{\lambda}} \approx e^{\frac{(-2)(0.55)(2\mu\text{m})}{66\text{nm}}} \approx e^{-33} \approx 0 \quad (6)$$

This shows that the exponential portion is very insignificant, thus this analysis will take $C = 1 + \frac{A_1 \lambda}{r}$.

Combining this correction factor with Stokes' Law gives the drag force:

$$\vec{F}_d = -\frac{6\pi\eta_a r}{\left(1 + \frac{A_1 \lambda}{r}\right)} \vec{v} \quad (7)$$

Where η_a is the viscosity of air, r is the droplet radius, $A_1 = 1.2557$ in air, $\lambda = 66\text{nm}$ for air, and \vec{v} is the droplet velocity. Note: λ varies between $64\text{nm} - 68\text{nm}$ in air. Here it is taken to be 66nm . The uncertainty of $\pm 2\text{nm}$ is insignificant relative to the uncertainty in \vec{v} and r .

In typical temperature ranges ($-20^\circ\text{C} - 400^\circ\text{C}$), the viscosity of air in SI unites ($\text{Pa} \cdot \text{s}$) is approximated by[7]:

$$\eta_a = 2.791 \times 10^{-7} \times T^{0.7355} \quad (8)$$

Where T is the temperature in kelvins.

2.2 Calculating Droplet Radius

Because many of the equations depend on the droplet radius, it is something that the analysis will need to focus on. In order to calculate the radius of each droplet, their paths will first be observed without an applied electric field. Gravity acting on the droplets means they will accelerate downwards ($-\hat{y}$). Because the force of drag opposes velocity (see Equation 7; $\vec{F}_d \propto -\vec{v}$), the force of drag will increase as it accelerates down until the drag force exactly opposes gravity. This is mechanical equilibrium, and results in a net zero acceleration. The velocity at which equilibrium is reached is called the falling terminal velocity, \vec{v}_f . The forces diagram is seen in Figure 4



Figure 4: Mechanical equilibrium without an \vec{E} field gives rise to a falling terminal velocity \vec{v}_f , as the only two forces are drag (\vec{F}_d) and gravity (\vec{F}_g).

Gravity and drag are the only dominant acting forces, and because they negate each other:

$$\vec{F}_d + \vec{F}_g = -\frac{6\pi\eta_a r}{\left(1 + \frac{A_1\lambda}{r}\right)}\vec{v}_f - \frac{4}{3}\pi r^3 \rho g \hat{y} = \vec{0} \quad (9)$$

Because all forces are in the $\pm\hat{y}$ direction ($\vec{v}_f = v_f \hat{y}$), one may only consider the \hat{y} components, so:

$$-\frac{6\pi\eta_a r}{\left(1 + \frac{A_1\lambda}{r}\right)}v_f - \frac{4}{3}\pi r^3 \rho g = 0 \Rightarrow \quad (10)$$

$$-\frac{3\eta_a v_f}{\left(1 + \frac{A_1\lambda}{r}\right)} = \frac{2}{3}r^2 \rho g \Rightarrow \quad (11)$$

$$r^2 + A_1\lambda r + \frac{9\eta_a v_f}{2\rho g} = 0 \quad (12)$$

The result is an equation quadratic in r , which can be solved using the quadratic equation. Neglecting the negative solution yields:

$$r = -\frac{A_1\lambda}{2} + \frac{1}{2}\sqrt{A_1^2\lambda^2 - \frac{18\eta_a v_f}{\rho g}} \quad (13)$$

Note: since the droplet is falling, $v_f < 0$.

2.3 Calculating Droplet Charge

To calculate the charge of a droplet after one has determined v_f and r , a voltage will be induced across the capacitor plates, creating a constant electric field \vec{E} in the region between the plates. Here the analysis will focus on negatively charged droplets for simplicity, so in the presence of a downward \vec{E} field the droplet will rise (see Equation 3). Like in Section 2.2, the presence of drag will result in the droplet's forces cancelling out, yielding a rising terminal velocity \vec{v}_r . Measuring this new terminal velocity and comparing it to the falling terminal velocity will allow us to calculate the charge Q of the droplet. The forces diagram is seen in Figure 5.

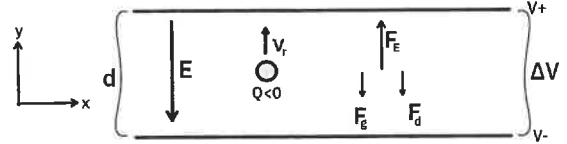


Figure 5: Mechanical equilibrium with an \vec{E} field gives rise to a rising terminal velocity \vec{v}_r , with the force of drag \vec{F}_d , force of gravity \vec{F}_g , and Coulomb force \vec{F}_E .

This mechanical equilibrium gives the following equation:

$$\vec{F}_E + \vec{F}_d + \vec{F}_g = -Q\frac{\Delta V}{d}\hat{y} - \kappa\vec{v}_r - mg\hat{y} = \vec{0} \Rightarrow \quad (14)$$

$$-Q\frac{\Delta V}{d} - \kappa v_r - mg = 0 \quad (15)$$

Where κ and m can be found as the coefficients of Equations 7 and 2. Knowledge can be combined by comparing this with the mechanical equilibrium equation of the scenario without an \vec{E} field from Section 2.2 (Equation 10). This gives two equations:

$$-\kappa\nu_f - mg = 0 \quad (16)$$

$$-Q \frac{\Delta V}{d} - \kappa\nu_r - mg = 0 \quad (17)$$

It is known from Equation 7 that κ depends on r , so reducing the dependence on it will reduce the amount of uncertainty that is introduced. Because the value of ν_f has already been determined for the droplet in question, one can solve the first equation for κ and substitute that into the second equation in the following way:

$$-\kappa = \frac{mg}{\nu_f} \Rightarrow \text{substitute} \Rightarrow \quad (18)$$

$$-Q \frac{\Delta V}{d} + mg \frac{\nu_r}{\nu_f} - mg = 0 \Rightarrow \quad (19)$$

$$Q = mg \frac{d}{\Delta V} \left(\frac{\nu_r}{\nu_f} - 1 \right) \quad (20)$$

After using Equation 2 to expand m , we're left with the following equation for the charge Q of a droplet:

$$Q = \frac{4}{3} \pi r^3 \rho g \frac{d}{\Delta V} \left(\frac{\nu_r}{\nu_f} - 1 \right) \quad (21)$$

Where d is the plate separation, ΔV is the voltage difference, ρ is the oil density, and g is the gravitational acceleration. Also note that $\nu_f < 0$ and $\nu_r > 0$, therefore we will have $Q < 0$. This is expected because we are focusing on droplets that rise in the presence of a downward electric field and because the gravitational force will be weaker than the Coulomb force.

3 Data Analysis & Uncertainty

3.1 Calculating ν and its Uncertainty

Central to the entire analysis thus far is the measurement of a droplet's terminal velocity, yet so far it has

been taken for granted. In practice one will actually be measuring the time Δt that it takes for a droplet to cover some vertical distance Δy , both of which will have some measurement error/uncertainty α_t, α_y .

After a measurement of Δt and Δy , the terminal velocity ν will be given by:

$$\nu = \frac{\Delta y}{\Delta t} \quad (22)$$

It is clear that ν is affected by both the uncertainty in Δy (α_y) and the uncertainty in Δt (α_t). To find the uncertainty in ν (α_ν), then, each uncertainty must be propagated individually to see its effect on ν and then combined[1].

The uncertainty in ν due to α_y is given by:

$$\alpha_\nu^y = \frac{1}{2} \left(\frac{\Delta y + \alpha_y}{\Delta t} - \frac{\Delta y}{\Delta t} \right) + \frac{1}{2} \left(\frac{\Delta y}{\Delta t} - \frac{\Delta y - \alpha_y}{\Delta t} \right) = \frac{\alpha_y}{\Delta t} \quad (23)$$

Similarly, α_t is propagated to give:

$$\alpha_\nu^t = -\frac{\Delta y \cdot \alpha_t}{\Delta t^2 - \alpha_t^2} \quad (24)$$

Like standard deviation, uncertainty does not combine linearly, rather it combines quadratically[1]. Therefore:

$$\alpha_\nu^2 = (\alpha_\nu^y)^2 + (\alpha_\nu^t)^2 \quad (25)$$

$$= \frac{\alpha_y^2}{\Delta t^2} + \frac{\Delta y^2 \cdot \alpha_t^2}{(\Delta t^2 - \alpha_t^2)^2} \quad (26)$$

Finally, this gives the total uncertainty in ν :

$$\alpha_\nu = \sqrt{\frac{\alpha_y^2}{\Delta t^2} + \frac{\Delta y^2 \cdot \alpha_t^2}{(\Delta t^2 - \alpha_t^2)^2}} \quad (27)$$

3.2 Combining Measurements

Once a drop has been selected for observation, as many measurements can be performed on it as are wanted, as long as the drop is kept from colliding with

either of the plates. This can be done by switching the plate voltage difference on and off (grounded), making the droplet rise or fall on demand.

Additionally, as long as the droplet's radius and charge do not change while the measurements are performed, the terminal velocity will remain the same. Therefore by combining multiple measurements of the same droplet (without changing r , Q), a better value for the terminal velocity can be ascertained.

Multiple measurements of the same terminal velocity can be combined in the following way[1]:

$$\nu = \frac{\sum_i \omega_i \nu_i}{\sum_i \omega_i} \quad (28)$$

$$\alpha_\nu = \frac{1}{\sqrt{\sum_i \omega_i}} \quad (29)$$

Where $\omega_i = \frac{1}{\alpha_{\nu_i}^2}$, and ν_i and α_{ν_i} are calculated from each individual measurement according to Equations 22, 27.

This same technique of combining multiple measurements will also be used for multiple calculations of r for each droplet, as well as combining the calculated value of e for multiple droplets into a single final value.

3.3 Uncertainty in r

Upon viewing Equation 13, it is clear that the calculated value of r depends on the falling terminal velocity ν_f . Because ν_f is the only experimentally determined value in the equation, it (α_{ν_f}) will be the dominant source of uncertainty.

Because of the complicated expression for r (Equation 13), uncertainty will not be propagated forwards and backwards like in Section 3.1. Instead, for small α_{ν_f} , the forward and backward propagation is the finite difference approximation to the derivative, thus:

$$\alpha_r \approx \frac{\partial r}{\partial \nu_f} \alpha_{\nu_f} \quad (30)$$

Computing this provides the equation for the uncertainty in droplet radius:

$$\alpha_r = - \frac{9\eta_a \alpha_{\nu_f}}{2\rho g \sqrt{A_1^2 \lambda^2 - \frac{18\eta_a \nu_f}{\rho g}}} \quad (31)$$

3.4 Uncertainty in Q

The uncertainty in droplet charge will be calculated in the same manner as uncertainty in droplet radius. However, Equation 21 demonstrates that there are now three dominant sources of uncertainty: α_r , α_{ν_r} , and α_{ν_f} . Because of this, the uncertainty for each must be calculated individually and then combined quadratically.

$$\alpha_Q^r \approx \frac{\partial Q}{\partial r} \alpha_r = 4\pi r^2 \rho g \frac{d}{\Delta V} \left(\frac{\nu_r}{\nu_f} - 1 \right) \alpha_r \quad (32)$$

$$\alpha_Q^{\nu_r} \approx \frac{\partial Q}{\partial \nu_r} \alpha_{\nu_r} = \frac{4}{3} \pi r^3 \rho g \frac{d}{\Delta V} \frac{\alpha_{\nu_r}}{\nu_f} \quad (33)$$

$$\alpha_Q^{\nu_f} \approx \frac{\partial Q}{\partial \nu_f} \alpha_{\nu_f} = -\frac{4}{3} \pi r^3 \rho g \frac{d}{\Delta V} \frac{\nu_r}{\nu_f^2} \alpha_{\nu_f} \quad (34)$$

Combining these individual uncertainties quadratically provides the equation for the uncertainty in droplet charge:

$$\alpha_Q = 4\pi r^2 \rho g \frac{d}{\Delta V} \sqrt{\left(\frac{\nu_r}{\nu_f} - 1 \right)^2 \alpha_r^2 + \frac{r^2}{9\nu_f^2} \alpha_{\nu_r}^2 + \frac{r^2 \nu_r^2}{9\nu_f^4} \alpha_{\nu_f}^2} \quad (35)$$

3.5 Determining e and its Uncertainty

After taking multiple falling and rising measurements for multiple charge states of multiple droplets, r can be calculated for each droplet and Q for each charge state. For this experiment two droplets were observed with seven different charge states each, with four falling and four rising measurements taken for each charge state.

With all of the previously mentioned values calculated, the value of e may be determined for each

individual droplet, where they can then all be combined into a single value of e as described in Section 3.2.

When the charge values for each droplet are plotted to an appropriate scale (10^{-19} C), clustering occurs due to the discrete nature of charge, with each charge state being associated with a small integer number of electrons n on the droplet. If sufficient charge states are measured, it can be assumed that the two clusters with the smallest separation in charge are separated by a single electron, or $\Delta n = 1$. Otherwise, intuition will need to be used to determine the smallest charge that divides each state's charge value. Each cluster can then be labeled with the appropriate value of electron quantity n .

This idea is more easily demonstrated by Figures 6 and 7, and is captured in the following equation:

$$Q_i = en_i \quad (36)$$

Where Q_i is the charge value, e is the elementary charge (the charge of an electron), and n_i is the number of electrons or elementary charges.

Equation 36 follows a linear relationship of the form $y_i = mx_i + b$. Because of this, one can perform a regression to fit the (n_i, Q_i) data to a straight line, the slope of which will be the droplet's value for e . A weighted least squares analysis yields the following equations[1]:

$$Q_i = en_i + b \quad (37)$$

$$e = \frac{\sum_i \omega_i \sum_i \omega_i n_i Q_i - \sum_i \omega_i n_i \sum_i \omega_i Q_i}{\Delta'} \quad (38)$$

$$b = \frac{\sum_i \omega_i n_i^2 \sum_i \omega_i Q_i - \sum_i \omega_i n_i \sum_i \omega_i n_i Q_i}{\Delta'} \quad (39)$$

$$\alpha_e = \sqrt{\frac{\sum_i \omega_i}{\Delta'}} \quad (40)$$

$$\alpha_b = \sqrt{\frac{\sum_i \omega_i n_i^2}{\Delta'}} \quad (41)$$

$$\omega_i = \frac{1}{\alpha_{Q_i}^2} \quad (42)$$

$$\Delta' = \sum_i \omega_i \sum_i \omega_i n_i^2 - (\sum_i \omega_i n_i)^2 \quad (43)$$

This regression fits the data to a straight line, giving e and α_e for a single droplet. These independent

values of e for each droplet can then be combined into a single value for e and α_e using the method described in Section 3.2.

The regression analysis for each droplet is shown in Figures 8 and 9, and yields the following two values for e :

$$e_1 = (-1.95 \pm 0.56) \times 10^{-19} \text{ C}$$

$$e_2 = (-1.51 \pm 0.28) \times 10^{-19} \text{ C}$$

The weighted combination of these (see Section 3.2) yields a final value for the elementary charge:

$$e = (-1.60 \pm 0.25) \times 10^{-19} \text{ C} \quad (44)$$

3.6 Measurement Error

The two original dominant sources of error are droplet timing and droplet displacement, as these are the two values that are being manually measured. These two errors are propagated through the analysis to the final value of e according to the diagram in Figure 10.

Consequently, the timing uncertainty and displacement uncertainty must be determined.

To determine our timing error, we used a stopwatch (the one used in our procedure) to measure our reaction speed by attempting to stop the timer at 5.00s. This simulates the need to stop the timer when a droplet crosses a gridline in the viewing microscope (Figure 1). The timing errors were recorded and then averaged to determine a value for α_t .

To determine displacement error, the oil-drop apparatus manual was consulted to determine the separation between minor gridlines in the viewing microscope. This value is taken to be α_y .

4 Detailed Methods

4.1 Equipment Setup

The following is a list of the equipment used during the experiment:

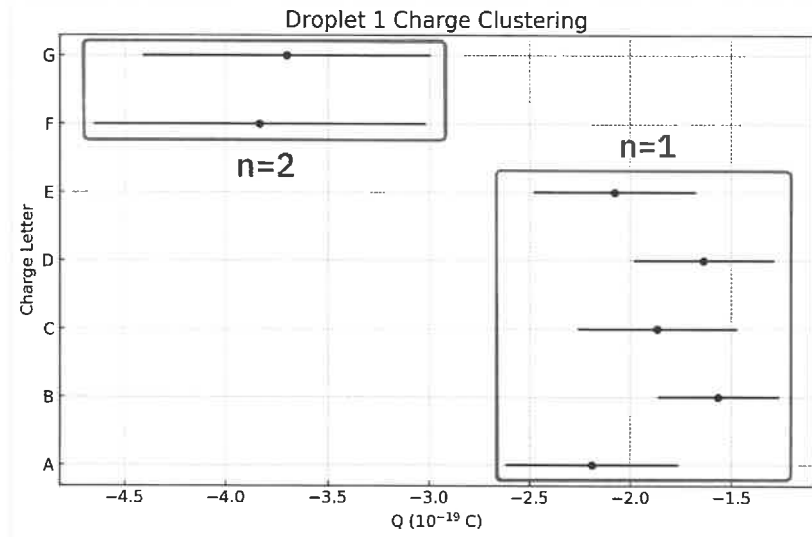


Figure 6: Plot of the calculated charges for droplet 1. The error bars correspond to the uncertainty in charge. This demonstrates the discreteness of charge with clusters, and allows the charge clusters to be labeled with their corresponding quantity of electrons n .

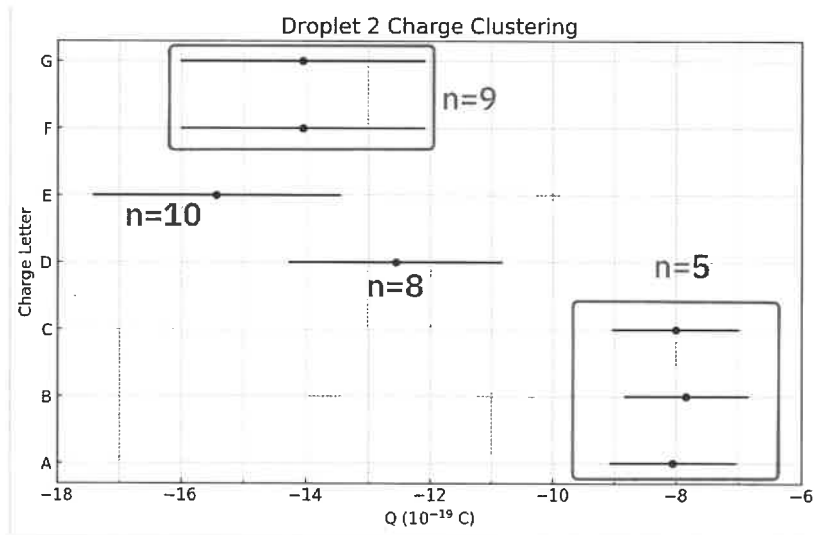


Figure 7: Plot of the calculated charges for droplet 2. The error bars correspond to the uncertainty in charge. This demonstrates the discreteness of charge with clusters, and allows the charge clusters to be labeled with their corresponding quantity of electrons n .

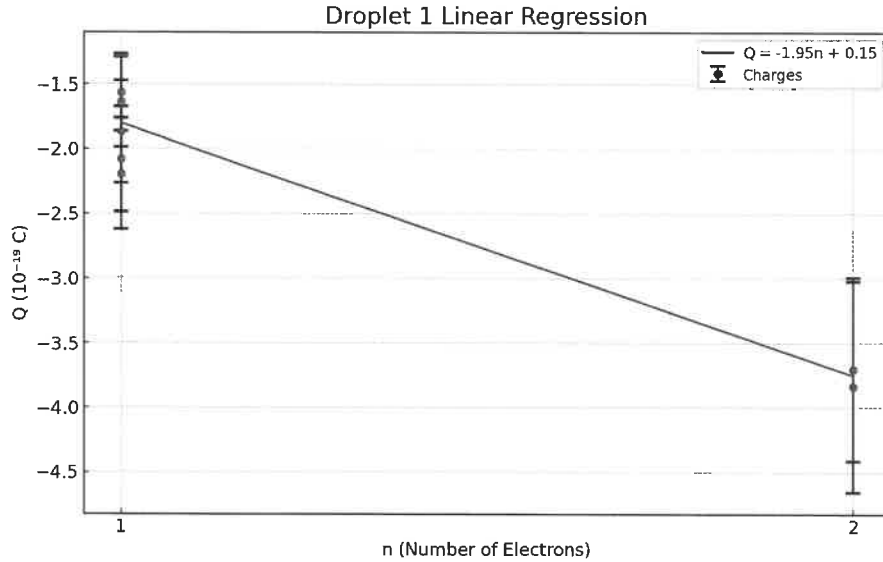
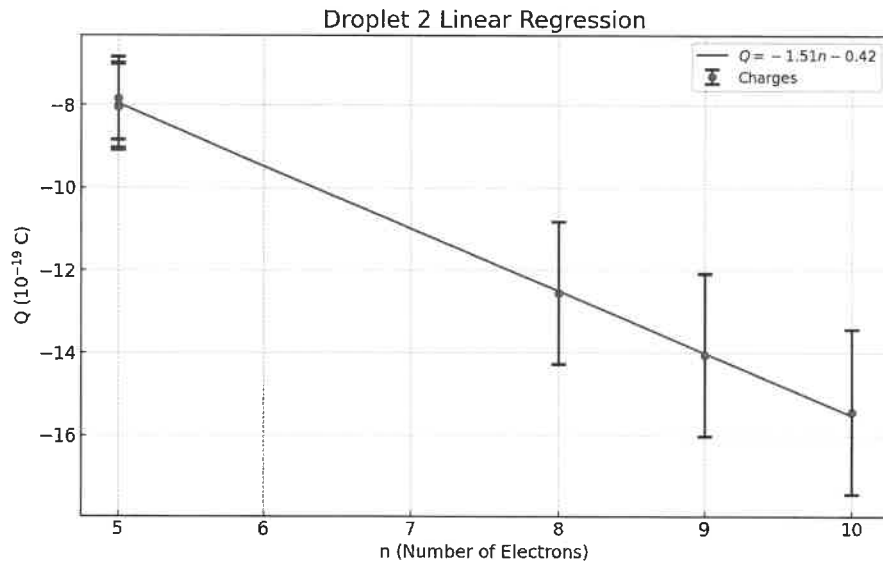


Figure 8: Plot of the linear regression for droplet 1. The error bars correspond to the charge uncertainty, and the value of the slope corresponds to the value of the elementary charge e . This regression yields $e_1 = (-1.95 \pm 0.56) \times 10^{-19} \text{ C}$. Because the charges for droplet 1 were only clustered around $n = 1$ and $n = 2$, the individual values may be difficult to distinguish.



Unlike in lab notebooks, in papers procedures are in paragraph form

If included, imbedded into procedure

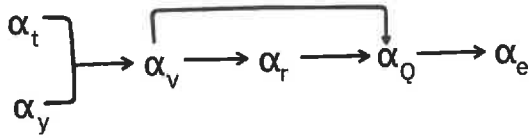


Figure 10: Diagram showing the flow of uncertainty through the analysis, starting from the initial uncertainty that arises due to measurement (α_t, α_y).

- Millikan oil drop apparatus: PASCO Model AP-8210 (#P00048158)
- Oil atomizer
- Non-volatile oil: Squib #5597 Mineral Oil ($\rho = 886 \text{ kg/m}^3$)
- 500V DC power supply
- Digital multi-meter
- Digital stopwatch

The apparatus setup can be seen in Figure 11. The viewing microscope contains an internal grid overlay that allows distances and positions of droplets to be measured.



Figure 11: The setup for the oil drop apparatus.

4.2 Procedure

A detailed description of the followed experimental steps, as outlined in Section 1.2, is found below:

1. Select a location free of drafts and vibrations.
2. Place the apparatus on a level surface. Use the included bubble level as a reference.
3. Disassemble the droplet viewing chamber. Measure and record the thickness of the plastic spacer separating the plates.
4. Clean any old oil off the inside of the chamber, the plates, and the plastic spacer.
5. Reassemble the droplet viewing chamber.
6. Focus the halogen filament.
7. Connect the DC power supply. Set it to deliver 500V between the plates.
8. Determine the chamber temperature using the multi-meter and apparatus thermistor.
9. Darken the room.
10. Ensure the apparatus background is dark.
11. Spray oil into the chamber with the atomizer.
12. Select a droplet that falls slowly enough for accurate measurement.
13. Focus the microscope onto that droplet. The droplet can be moved up and down by grounding and ungrounding the plates.
14. Get multiple (4) measurements of falling terminal velocity while the plates are grounded.
15. Get multiple (4) measurements of rising terminal velocity while plates are not grounded.
16. Use the apparatus ionization lever to give the droplet a new charge.
17. Do steps 14-16 multiple times (7) for the same droplet, resulting in multiple measured charge states of the droplet.

18. Do steps 11-17 for multiple (2) droplets.

An issue that was encountered multiple times was related to the leveling of the apparatus. Multiple suitable droplets were lost during measurement because they slowly drifted horizontally until out of view. This was fixed by re-leveling and ensuring that droplets fell vertically along the viewing grid before proceeding with measurement.

5 Conclusion

5.1 Improvements

Because droplet timing and displacement are the two largest sources of error, the improvements mentioned will focus on reducing their uncertainty.

The lowest-tech way of reducing timing error would be to use a high-speed camera to record the droplets as they are rising and falling in front of the background grid. This would take human reaction speed out of the loop.

The simplest way of reducing displacement error would be to use a focusing reticle (the grid) in the viewing microscope with much smaller gridline tick-marks. However, care must be taken to not select too small of droplets, as this would make molecular/Brownian motion a dominant source of uncertainty.

5.2 Summary

As seen in Equation 44, the experiment and analysis performed on two droplets gives the value of the elementary charge as $e = (-1.60 \pm 0.25) \times 10^{-19}$ C, whereas the currently accepted value is $e = -1.602177 \times 10^{-19}$ C. This results in the measured value having a Z-Score of 0.01, and therefore being in excellent agreement with conventional knowledge.

Additionally, the clustering seen in Figures 6 and 7 shows that charge is indeed quantized, such that there is a linear relationship between different charge states of an oil droplet (Figures 8, 9).

These results accomplish the goal of showing the quantization of charge, measuring the elementary charge e , and propagating uncertainty throughout

the analysis to determine the uncertainty associated with the final value of e .

References

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